

**Class - X Session 2022-23**  
**Subject - Mathematics (Basic)**  
**Sample Question Paper - 21**

**Time Allowed: 3 Hours**

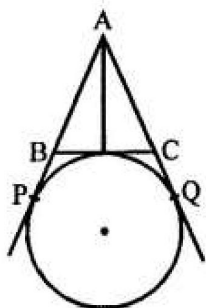
**Maximum Marks: 80**

### General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take  $\pi = 22/7$  wherever required if not stated.

## Section A

1. In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm then the length of AP is [1]



- a) 7.5 cm                      b) 15 cm
- c) 9 cm                         d) 10 cm
2. If the distance between the points (4, p) and (1,0) is 5, then the value of p is [1]
- a) 0                                  b) 4 only
- c) -4 only                        d)  $\pm 4$
3. The coordinates of the mid-point of the line segment joining the points ( $x_1, y_1$ ) and ( $x_2, y_2$ ) is given by [1]
- a)  $\left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$                       b)  $\left( \frac{x_1-x_2}{2}, \frac{y_1-y_2}{2} \right)$

c)  $\left(\frac{x_1-y_1}{2}, \frac{x_2-y_2}{2}\right)$

d)  $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$

4. If three coins are tossed simultaneously, then the probability of getting at least two heads, is [1]
- a)  $\frac{1}{2}$  b)  $\frac{3}{8}$
- c)  $\frac{1}{4}$  d)  $\frac{7}{4}$
5. If the point R(x, y) divides the join of P(x<sub>1</sub>, y<sub>1</sub>) and Q(x<sub>2</sub>, y<sub>2</sub>) internally in the given ratio m<sub>1</sub> : m<sub>2</sub>, then the coordinates of the point R are [1]
- a)  $\left(\frac{m_2x_1-m_1x_2}{m_1+m_2}, \frac{m_2y_1-m_1y_2}{m_1+m_2}\right)$  b)  $\left(\frac{m_2x_1-m_1x_2}{m_1-m_2}, \frac{m_2y_1-m_1y_2}{m_1-m_2}\right)$
- c)  $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$  d) None of these
6. Graphically, the pair of equations 6x - 3y + 10 = 0, 2x - y + 9 = 0 represents two lines which are [1]
- a) parallel b) Intersect at two points
- c) coincident d) intersect at a point
7. From the letters of the word MOBILE, a letter is selected. The probability that the letter is a vowel, is [1]
- a)  $\frac{3}{7}$  b)  $\frac{1}{6}$
- c)  $\frac{1}{2}$  d)  $\frac{1}{3}$
8. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal then the ratio of its radius and the slant height of the conical part is [1]
- a) 4 : 1 b) 1 : 4
- c) 1 : 2 d) 2 : 1
9. In a lottery, there are 8 prizes and 16 blanks. What is the probability of getting a prize? [1]
- a)  $\frac{1}{2}$  b) None of these
- c)  $\frac{1}{3}$  d)  $\frac{2}{3}$
10. If  $\alpha$  and  $\beta$  are the roots of  $ax^2 + bx + c = 0$ , then the wrong statement is [1]
- a)  $\alpha^2 + \beta^2 = \frac{b^2-2ac}{a^2}$  b)  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{c}$

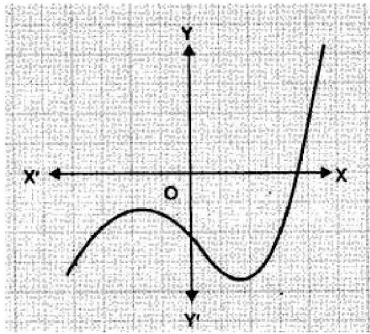


- c)  $\alpha + \beta = \frac{b}{a}$  d)  $\alpha\beta = \frac{c}{a}$
11.  $4x^2 - 2x - 3 = 0$  have [1]  
 a) Real roots b) Real and Distinct roots  
 c) No Real roots d) Real and Equal roots
12. If  $\sqrt{3} \tan \theta = 3 \sin \theta$ , then the value of  $\sin^2 \theta - \cos^2 \theta$  is [1]  
 a) 1 b)  $\frac{1}{2}$   
 c) 0 d)  $\frac{1}{3}$
13. Points (1, 0) and (-1, 0) lies on [1]  
 a) line  $x + y = 0$  b) y-axis  
 c) x-axis d) line  $x - y = 0$
14. 2.35 is [1]  
 a) an integer b) a rational number  
 c) an irrational number d) a natural number
15. Construction of cumulative frequency table is useful to determine [1]  
 a) mean b) all the three  
 c) median d) mode
16. If the length of a shadow of a tower is increasing, then the angle of elevation of the sun is [1]  
 a) neither increasing nor decreasing b) zero  
 c) decreasing d) increasing
17. The HCF and the LCM of 12, 21, 15 respectively are: [1]  
 a) 3, 140 b) 420, 3  
 c) 12, 420 d) 3, 420
18. **Assertion (A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162 [1]  
**Reason:** If a, b are two positive integers, then  $\text{H.C.F.} \times \text{L.C.M.} = a \times b$   
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.  
 c) A is true but R is false. d) A is false but R is true.









### Section C

26. Evaluate:  $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$  [3]

27. Prove that  $7\sqrt{5}$  is irrational. [3]

OR

In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

28. Form the pair of linear equations for the problem and find its solution by substitution method: [3]

The difference between the two numbers is 26 and one number is three times the other.

29. O is the centre of a circle of radius 5 cm. T is a point such that  $OT = 13$  cm and  $OT$  intersects the circle at E. If AB is the tangent to the circle at E. Find length of AB. [3]

OR

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

30. Diagonal AC and BD of a trapezium ABCD with  $AB \parallel DC$  intersect each other at point O. Using a similarity criterion for two triangles, show that  $\frac{OA}{OC} = \frac{OB}{OD}$ . [3]

31. Two boats approach a light house in mid-sea from opposite directions. The angles of elevations of the top of the lighthouse from two boats are  $30^\circ$  and  $45^\circ$  respectively. If the distance between two boats is 100 m, find the height of the lighthouse. [3]

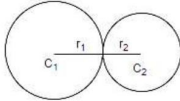
### Section D

32. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train. [5]

OR

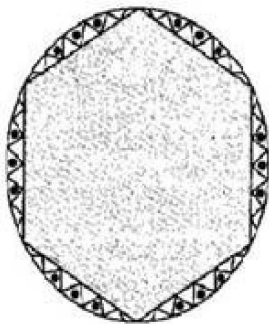
If the price of a book is reduced by ₹5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

33. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is  $130\pi$  and the distance between their centres is 14 m. Find the radii of the circles. What value is depicted by the farmers? [5]

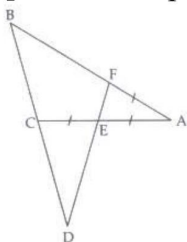


OR

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm<sup>2</sup>. (use  $\sqrt{3} = 1.7$ )



34. In the given figure, line segment DF intersect the side AC of a triangle  $\triangle ABC$  at the point E such that E is the mid-point of CA and  $\angle AEF = \angle AFE$ . Prove that:  $\frac{BD}{CD} = \frac{BF}{CF}$ . [5]  
 [Hint: Take point G on AB such that  $CG \parallel DF$ .]



35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. [5]

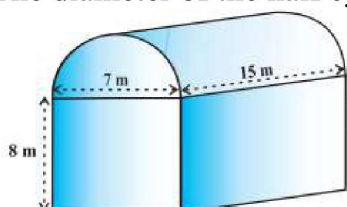
Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7

## Section E

36. Read the text carefully and answer the questions:

[4]

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were  $15\text{ m} \times 7\text{ m} \times 8\text{ m}$ . The diameter of the half cylinder was  $7\text{ m}$  and length was  $15\text{ m}$ .



- Find the volume of the air that the shed can hold.
- If the industry requires machinery which would occupy a total space of  $300\text{ m}^3$  and there are 20 workers each of whom would occupy  $0.08\text{ m}^3$  space on an average, how much air would be in the shed when it is working?
- Find the surface area of the cuboidal part.

OR

Find the surface area of the cylindrical part.

37. Read the text carefully and answer the questions:

[4]

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



- If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it
- Find the total money he saved.
- How much money Akshar saves in 10 days?

OR

How many coins are there in piggy bank on 15th day?

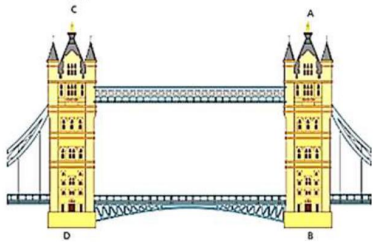
38. Read the text carefully and answer the questions:

[4]

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two

bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was  $60^\circ$  and  $30^\circ$  respectively.



- (i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.
- (ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?
- (iii) Find the distance between Neeta and top of tower AB?

**OR**

Find the distance between Neeta and top tower CD?

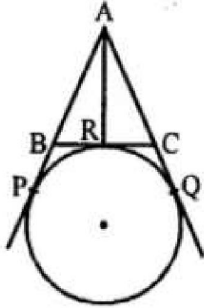


## SOLUTION

### Section A

1. (a) 7.5 cm

**Explanation:** In the given figure, AP, AQ and BC are tangents to the circle.



AB = 5 cm, AC = 6 cm, BC = 4 cm

Length of AP = ?

BP and BR are the tangents to the circle.

BP = BR

Similarly, CR and CQ are tangents

CR = CQ

Similarly, AP and AQ are tangents

AP = AQ

AP = AB + BP = AB + BR

AQ = AC + CQ = AC + CR

AP + AQ = AB + BR + AC + CR = AB + BR + CR + AC

AP + AP = AB + BC + AC

2AP = 5 + 4 + 6 = 15 cm

AP =  $\frac{15}{2}$  = 7.5 cm

2. (d)  $\pm 4$

**Explanation:** Distance between (4, p) and (1, 0) = 5

$$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$$

$$\Rightarrow \sqrt{(1 - 4)^2 + (0 - p)^2} = 5$$

$$\sqrt{(-3)^2 + (-p)^2} = 5$$

Squaring, both sides

$$(-3)^2 + (-p)^2 = (5)^2 \Rightarrow 9 + p^2 = 25$$

$$\Rightarrow p^2 = 25 - 9 = 16$$

$$\therefore p = \pm \sqrt{16} = \pm 4$$

3. (a)  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

**Explanation:** we know that the midpoint formula =  $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

The coordinates of the mid-point of the line segment joining the points (x<sub>1</sub>, y<sub>1</sub>) and

(x<sub>2</sub>, y<sub>2</sub>) is given by  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ .

4. (a)  $\frac{1}{2}$

**Explanation:** Possible outcomes of tossing three coins are:  
(HHH), (HHT), (HTH), (THH), (TTT), (TTH), (THT), (HTT)  
here H and T are denoted for Head and Tail.

Total outcomes = 8

no. of outcomes with at least two heads = 4

$$\therefore \text{required probability} = \frac{4}{8} = \frac{1}{2}$$

5. (c)  $\left( \frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2} \right)$

**Explanation:** If the point R(x, y) divides the join of P(x<sub>1</sub>, y<sub>2</sub>) and Q(x<sub>2</sub>, y<sub>2</sub>) internally in the given ratio m<sub>1</sub> : m<sub>2</sub>,

then the coordinates of the point R are  $\left( \frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2} \right)$

6. (a) parallel

**Explanation:** Given: a<sub>1</sub> = 6, a<sub>2</sub> = 2, b<sub>1</sub> = -3, b<sub>2</sub> = -1, c<sub>1</sub> = 10 and c<sub>2</sub> = 9

$$a_1 = 6, a_2 = 2, b_1 = -3, b_2 = -1, c_1 = 10 \text{ and } c_2 = 9$$

$$\text{Here } \frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$$

$$\text{but } \frac{c_1}{c_2} = \frac{10}{9}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the lines are parallel.

7. (c)  $\frac{1}{2}$

**Explanation:** No. of total letters in the word MOBILE = 6

No. of vowels = {O, I, E}, i, e = 3

$$\text{Probability of being a vowel} = \frac{3}{6} = \frac{1}{2}$$

8. (c) 1 : 2

$$\text{Explanation: } 2\pi r^2 = \pi r l \Rightarrow \frac{r}{l} = \frac{1}{2}$$

9. (c)  $\frac{1}{3}$

**Explanation:** Total number of lottery tickets = 8 + 16 = 24.

Number of prizes = 8.

$$\therefore P(\text{getting a prize}) = \frac{8}{24} = \frac{1}{3}$$

10. (c)  $\alpha + \beta = \frac{b}{a}$

**Explanation:** If  $\alpha$  and  $\beta$  are the roots of  $ax^2 + bx + c = 0$ ,  
then  $\alpha + \beta = \frac{-b}{a}$

11. (b) Real and Distinct roots

$$\text{Explanation: } D = b^2 - 4ac$$

$$D = (-2)^2 - 4 \times 4 \times (-3)$$

$$D = 4 + 48$$

$$D = 52$$

D > 0. Hence Real and Distinct roots.

12. (d)  $\frac{1}{3}$

**Explanation:** Given:  $\sqrt{3} \tan \theta = 3 \sin \theta$

$$\Rightarrow \sqrt{3} \frac{\sin \theta}{\cos \theta} = 3 \sin \theta$$

$$\Rightarrow \frac{\sqrt{3}}{3} = \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{3}}$$

$$\text{And } \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$$

$$\therefore \sin^2 \theta - \cos^2 \theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

13. (c) x-axis

**Explanation:** Since the ordinates of given points are 0. Therefore, points lie on x-axis.

14. (b) a rational number

**Explanation:** It can be expressed in  $\frac{p}{q}$  form

$$2.35 = \frac{235}{100}$$

so, 2.35 is a rational number

15. (c) median

**Explanation:** A cumulative frequency distribution is the sum of the class and all classes below it in a frequency distribution. Construction of cumulative frequency table is useful to determine Median.

16. (c) decreasing

**Explanation:**

If the elevation moves towards the tower, it is increasing and if its elevation moves away from the tower, it decreases. Hence if the shadow of a tower is increasing, then the angle of elevation of the sun is not increasing.

17. (d) 3, 420

**Explanation:** We have,

$$12 = 2 \times 2 \times 3$$

$$21 = 3 \times 7$$

$$15 = 5 \times 3$$

$$\text{HCF} = 3$$

$$\text{and L.C.M} = 2 \times 2 \times 3 \times 5 \times 7 = 420$$

18. (d) A is false but R is true.

$$\text{Explanation: } \frac{3072}{16} = 192 \neq 162$$

19. (d) 6

$$\text{Explanation: For non-zero solution } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{k} = \frac{5}{10} = \frac{0}{0}$$

$$\text{Taking, } \frac{3}{k} = \frac{5}{10} \Rightarrow k = \frac{3 \times 10}{5} = 6$$

20. (d) A is false but R is true.

**Explanation:** Similar triangles are not always congruent.

### Section B

21. i. Total number of favourable outcomes = 20

$$\text{Probability of the event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$\text{Number of favourable outcomes} = 4$$

$$\text{Hence P (getting a defective bulb)} = \frac{4}{20} = \frac{1}{5}$$

ii. Now total number of favourable outcomes = 20 - 1 = 19

$$\text{Number of favourable outcomes} = 19 - 4 = 15$$

Hence P (getting a non-defective bulb) =  $\frac{15}{19}$

22. Let the present age of Aftab and his daughter be  $x$  and  $y$  years respectively. Then, the pair of linear equations that represent the situation is

$$x - 7 = 7(y - 7), \text{ i.e., } x - 7y + 42 = 0 \dots(1)$$

$$\text{and } x + 3 = 3(y + 3), \text{ i.e., } x - 3y = 6 \dots(2)$$

from equation (2), we get  $x = 3y + 6$

By putting this value of  $x$  in equation (1), we get

$$(3y + 6) - 7y + 42 = 0,$$

$$\text{i.e., } -4y = -48, \text{ which gives } y = 12$$

Again by putting this value of  $y$  in equation (2), we get

$$x = 3 \times 12 + 6 = 42$$

So, the present age of Aftab and his daughter are 42 and 12 years respectively.

OR

Let length of rectangular garden =  $x$  metres

and width of rectangular garden =  $y$  metres

According to given conditions, perimeter = 36 m

$$\frac{1}{2}(2x + 2y) = 36$$

$$\Rightarrow x + y = 36 \dots\dots(i)$$

$$\text{and } x = y + 4$$

$$\Rightarrow x - y = 4 \dots\dots(ii)$$

Adding eq. (i) and (ii),

$$2x = 40 \Rightarrow x = 20 \text{ m}$$

Subtracting eq. (ii) from eq. (i),

$$2y = 32 \Rightarrow y = 16 \text{ m}$$

Hence, length = 20 m and width = 16 m

23. Let the coordinates of the required point be  $(x, y)$ . Then,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$= \frac{(2)(4) + (3)(-1)}{2 + 3}$$

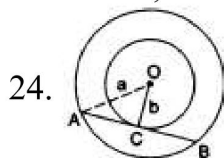
$$= \frac{8 - 3}{5} = \frac{5}{5} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{(2)(-3) + (3)(7)}{2 + 3}$$

$$= \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Hence, the required point is  $(1, 3)$ .



Let  $O$  be the common centre of the two circles

and  $AB$  be the chord of the larger circle which touches the smaller circle at  $C$ .

Join  $OA$  and  $OC$ .

Then,  $OA = a$  and  $OC = b$ .

Now,  $OC \perp AB$  and  $OC$  bisects  $AB$  [ $\because$  the chord of the larger circle touching the smaller circle, is bisected at the point of contact].

In right  $\triangle ACO$ , we have

$$OA^2 = OC^2 + AC^2 \text{ [by Pythagoras' theorem]}$$

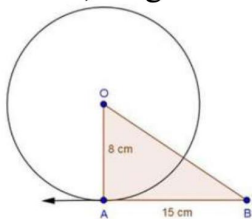
$$\Rightarrow AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}.$$

$$\therefore AB = 2AC = 2\sqrt{a^2 - b^2} \text{ } [\because C \text{ is the midpoint of } AB]$$

$$\text{i.e., required length of the chord } AB = 2\sqrt{a^2 - b^2}$$

OR

Since, tangent at a point on a circle is perpendicular to the radius through the point.



Therefore, OA is perpendicular to AB.

In right triangle OAB, We have

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow OB^2 = 8^2 + 15^2$$

$$= 64 + 225$$

$$= 289$$

$$\Rightarrow OB^2 = 289$$

$$\Rightarrow OB^2 = (17)^2$$

$$\Rightarrow OB = 17 \text{ cm.}$$

25. The number of zeroes is 1 as the graph intersects the x-axis at one point only.

### Section C

26. We have  $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

after putting values, we get

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$\text{Rationalise it, we get}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16}$$

$$= \frac{43-24\sqrt{3}}{11}$$

$$= \frac{43-24\sqrt{3}}{11}$$



27. We can prove  $7\sqrt{5}$  irrational by contradiction.

Let us suppose that  $7\sqrt{5}$  is rational.

It means we have some co-prime integers  $a$  and  $b$  ( $b \neq 0$ ) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b} \dots\dots(1)$$

R.H.S of (1) is rational but we know that  $\sqrt{5}$  is irrational.

It is not possible which means our supposition is wrong.

Therefore,  $7\sqrt{5}$  cannot be rational.

Hence, it is irrational.

OR

The number of participants in each room must be the HCF of 60, 84 and 108.

In order to find the HCF of 60, 84 and 108, we first find the HCF of 60 and 84 by Euclid's division algorithm:

2	60	84	1
	48	60	
	12	24	2
	(HCF)	24	
		0	
		(Remainder)	

Clearly, HCF of 60 and 84 is 12

Now, we find the HCF of 12 and 108

12	108	9
(HCF)	108	
	0	
	(Remainder)	

Clearly, HCF of 12 and 108 is 12. Hence, the HCF of 60,84 and 108 is 12.

Therefore, in each room maximum 12 participants can be seated.

We have,

$$\text{Total number of participants} = 60 + 84 + 108 = 252$$

$$\therefore \text{Number of rooms required} = \frac{252}{12} = 21.$$

28. Let the two numbers be  $x$  and  $y$  ( $x > y$ ) then, according to the question, the pair of linear equations formed is:

$$x - y = 26 \dots\dots\dots(1)$$

$$x = 3y \dots\dots\dots(2)$$

Substitute the value of  $x$  from equation (2) in equation (1), we get

$$3y - y = 26$$

$$\Rightarrow 2y = 26$$

$$\Rightarrow y = \frac{26}{2}$$

$$\Rightarrow y = 13$$

Substituting this value of  $y$  in equation (2), we get

$$x = 3(13) = 39$$

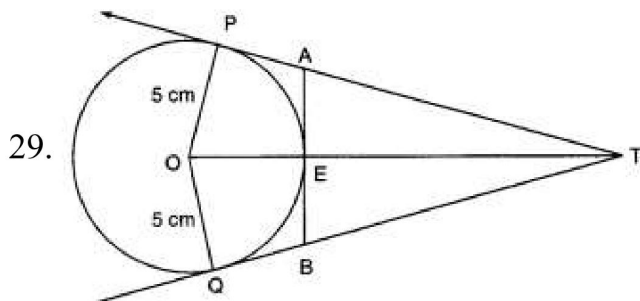
Hence, the required numbers are 39 and 13.

verification: Substituting  $x = 39$  and  $y = 13$ , we find that both the equation (1) and (2) are satisfied as shown below:

$$x - y = 39 - 13 = 26$$

$$3y = 3(13) = 39 = x.$$

This verifies the solution.



Clearly  $\angle OPT = 90^\circ$

Applying Pythagoras in  $\triangle OPT$ , we have

$$OT^2 = OP^2 + PT^2$$

$$\Rightarrow 13^2 = 5^2 + PT^2$$

$$\Rightarrow PT^2 = 169 - 25 = 144$$

$$\Rightarrow PT = 12 \text{ cm}$$

Since lengths of tangents drawn from a point to a circle are equal. Therefore,

$$AP = AE = x(\text{say})$$

$$\Rightarrow AT = PT - AP = (12 - x)\text{cm}$$

Since AB is the tangent to the circle E. Therefore,  $OE \perp AB$

$$\Rightarrow \angle OEA = 90^\circ$$

$$\Rightarrow \angle AET = 90^\circ [\text{Applying Pythagoras Theorem in } \triangle AET]$$

$$\Rightarrow (12 - x)^2 = x^2 + (13 - 5)^2$$

$$\Rightarrow 144 - 24x + x^2 = x^2 + 64$$

$$\Rightarrow 24x = 80$$

$$\Rightarrow x = \frac{10}{3} \text{ cm}$$

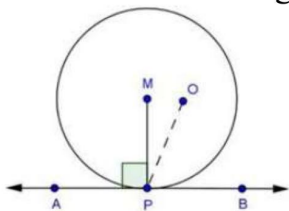
$$\text{Similarly, } BE = \frac{10}{3} \text{ cm}$$

$$\therefore AB = AE + BE = \left(\frac{10}{3} + \frac{10}{3}\right) \text{ cm}$$

$$= \frac{20}{3} \text{ cm}$$

OR

Let APB be the tangent and take O as centre of the circle.



Let us suppose that  $MP \perp AB$  does not pass through the centre.

Then,

$\angle OPA = 90^\circ$  [ $\because$  Tangent is perpendicular to the radius of circle]

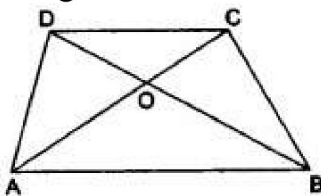
But  $\angle MPA = 90^\circ$  [Given]

$\therefore \angle OPA = \angle MPA$

This is only possible when point O and point M coincide with each other.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

30.



Given A trapezium ABCD in which  $AB \parallel DC$ . The diagonals AC and BD intersect at O.

To Prove In  $\triangle OAB$  and  $\triangle OCD$ , we have

$\angle OAB = \angle OCD$  [alternate angles, since  $AB \parallel DC$ ]

and  $\angle OBA = \angle ODC$  [alternate angles, since  $AB \parallel DC$ ]

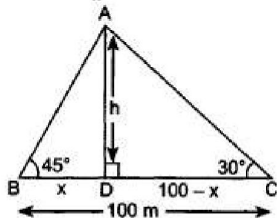
$\therefore \triangle OAB \sim \triangle OCD$  [by AA-similarity].

Hence,  $\frac{OA}{OC} = \frac{OB}{OD}$

31. In right  $\triangle ADB$ ,

$h = x$

$\Rightarrow \frac{h}{x} = \tan 45^\circ \dots (i)$



Now in rt.  $\triangle ADC$

$\frac{h}{100-x} = \tan 30^\circ$

Solve for h and x.

$\Rightarrow \frac{h}{100-x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h = 100 - x$

$\Rightarrow \sqrt{3}x = 100 - x$  [Using eq.(i)]

$\Rightarrow (\sqrt{3} + 1)x = 100 \Rightarrow x = \frac{100}{\sqrt{3}+1}$

$\Rightarrow x = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\Rightarrow x = \frac{100(\sqrt{3}-1)}{2} = 50(\sqrt{3} - 1)\text{m}$

$\therefore h = \text{height of lighthouse} = 50(\sqrt{3} - 1)\text{m}$

Section D

32. Let the usual speed of train be  $x$  km/hr

$$\frac{300}{x} - \frac{300}{x+5} = 2$$

$$300(x+5 - x) = 2x(x+5)$$

$$150(5) = x^2 + 5x$$

$$750 = x^2 + 5x$$

$$\text{or, } x^2 + 5x - 750 = 0$$

$$\text{or, } x^2 + 30x - 25x - 750 = 0$$

$$\text{or, } (x + 30)(x - 25) = 0$$

$$\text{or, } x = -30 \text{ or } x = 25$$

Since, speed cannot be negative.

$$\therefore x \neq -30, x = 25 \text{ km/hr}$$

$\therefore$  Speed of train = 25 km/hr

OR

Let the original list price be Rs  $x$

$$\therefore \text{No. of books bought for Rs 300} = \frac{300}{x}$$

Reduced list price of the book = Rs  $(x - 5)$

$$\text{No. of books bought for Rs 300} = \frac{300}{x-5}$$

According to question,

$$\frac{300}{x-5} - \frac{300}{x} = 5$$

$$\Rightarrow \frac{300x - 300x + 1500}{x^2 - 5x} = 5$$

$$\Rightarrow x^2 - 5x = 300 \Rightarrow x^2 - 5x - 300 = 0$$

$$\Rightarrow x^2 - 20x + 15x - 300 = 0$$

$$\Rightarrow (x - 20)(x + 15) = 0$$

$$\Rightarrow x = 20 \text{ or } x = -15$$

$$\Rightarrow x = 20$$

The negative sign is rejected.

Therefore  $x = 20$

Therefore the original price list is Rs. 20

33. Let the radii of the two circular plots be  $r_1$  and  $r_2$ , respectively.

Then,  $r_1 + r_2 = 14$  [ $\because$  Distance between the centres of two circular plots = 14 cm, given]....(i)

Also, Sum of Areas of the plots =  $130\pi$

$$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130 \dots (ii)$$

Now, from equation (i) and equation (ii),

$$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$$

$$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$$

$$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$$

Solving the quadratic equation we get,

$$r_2 = 3 \text{ or } r_2 = 11,$$

but from figure it is clear that,  $r_1 > r_2$

$$\therefore r_1 = 11 \text{ cm and } r_2 = 3 \text{ cm}$$

The value depicted by the farmers are of cooperative nature and mutual understanding.

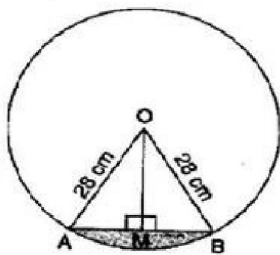
OR

$$r = 28 \text{ cm and } \theta = \frac{360}{6} = 60^\circ$$

$$\text{Area of minor sector} = \frac{\theta}{360} \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$$

$$= 410.67 \text{ cm}^2$$

For, Area of  $\triangle AOB$ ,



Draw  $OM \perp AB$ .

In right triangles OMA and OMB,

$OA = OB$  [Radii of same circle]

$OM = OM$  [Common]

$\therefore \triangle OMA \cong \triangle OMB$  [RHS congruency]

$\therefore AM = BM$  [By CPCT]

$$\Rightarrow AM = BM = \frac{1}{2} AB \text{ and } \angle AOM = \angle BOM = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60^\circ = 30^\circ$$



In right angled triangle OMA,  $\cos 30^\circ = \frac{OM}{OA}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$$

$$\Rightarrow OM = 14\sqrt{3} \text{ cm}$$

Also,  $\sin 30^\circ = \frac{AM}{OA}$

$$\Rightarrow \frac{1}{2} = \frac{AM}{28}$$

$$\Rightarrow AM = 14 \text{ cm}$$

$$\Rightarrow 2 AM = 2 \times 14 = 28 \text{ cm}$$

$$\Rightarrow AB = 28 \text{ cm}$$

$$\therefore \text{Area of } \triangle AOB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 28 \times 14\sqrt{3} = 196\sqrt{3} = 196 \times 1.7 = 333.2 \text{ cm}^2$$

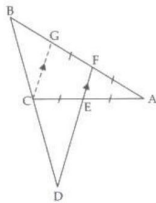
$$\therefore \text{Area of minor segment} = \text{Area of minor sector} - \text{Area of } \triangle AOB = 410.67 - 333.2 = 77.47 \text{ cm}^2$$

$$\therefore \text{Area of one design} = 77.47 \text{ cm}^2$$

$$\therefore \text{Area of six designs} = 77.47 \times 6 = 464.82 \text{ cm}^2$$

$$\text{Cost of making designs} = 464.82 \times 0.35 = \text{Rs. } 162.68$$

34.



To Prove:  $\frac{BD}{CD} = \frac{BF}{CE}$

Construction: Draw  $CG \parallel EF$ .

Proof: In  $\triangle AGC$   $CG \parallel EF$

$\therefore$  E is the mid point of AC

$\therefore$  F will be the mid point of AG.

$$\Rightarrow FG = FA$$

But,  $EC = EA = AF$  [Given]

$$\therefore FG = FA = EA = EC \dots (i)$$

In  $\triangle BCG$  and  $\triangle BDF$

$EF \parallel CG$ . (By construction)

$$\therefore \frac{BC}{CD} = \frac{BG}{GF} \text{ [By BPT]}$$

$$\Rightarrow \frac{BC}{CD} + 1 = \frac{BG}{GF} + 1$$

$$\Rightarrow \frac{BC+CD}{CD} = \frac{BG+GF}{GF}$$

$$\Rightarrow \frac{BD}{CD} = \frac{BF}{GF}$$

But,  $FG = CE$  [From (i)]

$\Rightarrow \frac{BD}{CD} = \frac{BF}{CE}$   
Hence, proved.

35.

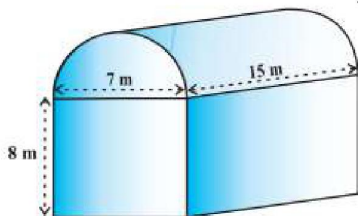
Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,  
 $N = \sum f_i = 100$   
 $\Rightarrow 76 + x + y = 100$   
 $\Rightarrow x + y = 24$   
 It is given that the median is 525. Clearly, it lies in the class 500 - 600  
 $\therefore l = 500, h = 100, f = 20, F = 36 + x$  and  $N = 100$   
 Now, Median =  $l + \frac{\frac{N}{2} - F}{f} \times h$   
 $\Rightarrow 525 = 500 + \frac{50 - (36 + x)}{20} \times 100$   
 $\Rightarrow 525 - 500 = (14 - x)5$   
 $\Rightarrow 25 = 70 - 5x$   
 $\Rightarrow 5x = 45$   
 $\Rightarrow x = 9$   
 Putting  $x = 9$  in  $x + y = 24$ , we get  $y = 15$   
 Hence,  $x = 9$  and  $y = 15$

### Section E

36. Read the text carefully and answer the questions:

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m. The diameter of the half cylinder was 7 m and length was 15 m.



(i) Total volume = volume of cuboid +  $\frac{1}{2} \times$  volume of cylinder.

For cuboidal part we have



length = 15 m, breadth = 7 m and height = 8 m

∴ Volume of cuboidal part =  $l \times b \times h = 15 \times 7 \times 8 \text{ m}^3 = 840 \text{ m}^3$

Clearly,

$r$  = Radius of half-cylinder =  $\frac{1}{2}$  (Width of the cuboid) =  $\frac{7}{2} \text{ m}$

and,  $h$  = Height (length) of half-cylinder = Length of cuboid = 15 m

∴ Volume of half-cylinder =  $\frac{1}{2} \pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15 \text{ m}^3$   
 $= \frac{1155}{4} \text{ m}^3 = 288.75 \text{ m}^3$

Thus the volume of the air that the shed can hold =  $(840 + 288.75) \text{ m}^3 = 1128.75 \text{ m}^3$

(ii) Total space occupied by 20 workers =  $20 \times 0.08 \text{ m}^3 = 1.6 \text{ m}^3$

Total space occupied by the machinery =  $300 \text{ m}^3$

∴ Volume of the air inside the shed when there are machine and workers inside it

$= (1128.75 - 1.6 - 300) \text{ m}^3$

$= (1128.75 - 301.6) \text{ m}^3 = 827.15 \text{ m}^3$

Hence, volume of air when there are machinery and workers is 827.15

(iii) Given for the cuboidal part

length  $L$  = 15 m, Width  $B$  = 7 m, Height = 8 m

Surface area of the cuboidal part

$= 2(LB + BH + HL)$

$= 2(15 \times 7 + 7 \times 8 + 8 \times 15)$

$= 2(105 + 56 + 120) = 2 \times 281 = 562 \text{ m}^2$

OR

For the cylindrical part  $r$  = 3.5 cm and  $l$  = 15 m

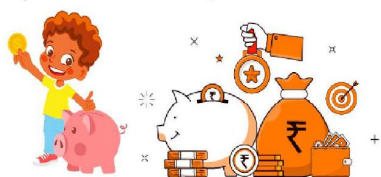
Thus the surface area of the cylindrical part

$= \frac{1}{2}(2\pi rl) = 3.14 \times 3.5 \times 15$

$= 164.85 \text{ m}^2$

### 37. Read the text carefully and answer the questions:

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) Child's Day wise are,

$\frac{5}{1 \text{ coin}}, \frac{10}{2 \text{ coins}}, \frac{15}{3 \text{ coins}}, \frac{20}{4 \text{ coins}}, \frac{25}{5 \text{ coins}}, \dots$  to  $\frac{n \text{ days}}{n \text{ coins}}$

We can have at most 190 coins

i.e.,  $1 + 2 + 3 + 4 + 5 + \dots$  to  $n$  term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n - 1)1] = 190$

$$\Rightarrow n(n+1) = 380 \Rightarrow n^2 + n - 380 = 0$$

$$\Rightarrow (n+20)(n-19) = 0 \Rightarrow (n+20)(n-19) = 0$$

$$\Rightarrow n = -20 \text{ or } n = 19 \Rightarrow n = -20 \text{ or } n = 19$$

But number of coins cannot be negative

$\therefore n = 19$  (rejecting  $n = -20$ )

So, number of days = 19

(ii) Total money she saved =  $5 + 10 + 15 + 20 + \dots = 5 + 10 + 15 + 20 + \dots$  upto 19 terms

$$= \frac{19}{2} [2 \times 5 + (19 - 1)5]$$

$$= \frac{19}{2} [100] = \frac{1900}{2} = 950$$

and total money she saved = ₹950

(iii) Money saved in 10 days

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{10} = \frac{10}{2} [2 \times 5 + (10 - 1) \times 5]$$

$$\Rightarrow S_{10} = 5 [10 + 45]$$

$$\Rightarrow S_{10} = 275$$

Money saved in 10 days = ₹275

OR

Number of coins in piggy bank on 15th day

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{15} = \frac{15}{2} [2 \times 5 + (15 - 1) \times 5]$$

$$\Rightarrow S_{15} = \frac{15}{2} [2 + 14]$$

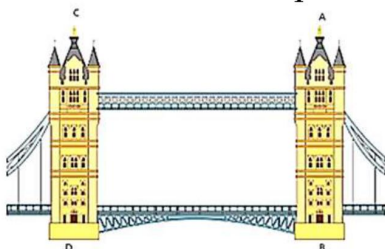
$$\Rightarrow S_{15} = 120$$

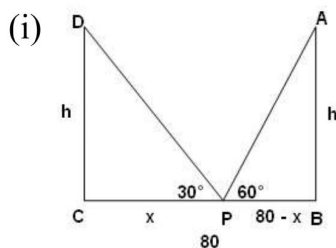
So, there are 120 coins on 15th day.

### 38. Read the text carefully and answer the questions:

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was  $60^\circ$  and  $30^\circ$  respectively.





Suppose AB and CD are the two towers of equal height  $h$  m. BC be the 80 m wide road. P is any point on the road. Let CP be  $x$  m, therefore BP =  $(80 - x)$ .

Also,  $\angle APB = 60^\circ$  and  $\angle DPC = 30^\circ$

In right angled triangle DCP,

$$\tan 30^\circ = \frac{CD}{CP}$$

$$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{x}{\sqrt{3}} \dots\dots(i)$$

In right angled triangle ABP,

$$\tan 60^\circ = \frac{AB}{BP}$$

$$\Rightarrow \frac{h}{80-x} = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3}(80 - x)$$

$$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$$

$$\Rightarrow x = 3(80 - x)$$

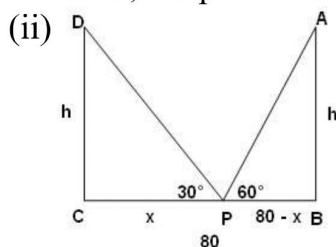
$$\Rightarrow x = 240 - 3x$$

$$\Rightarrow x + 3x = 240$$

$$\Rightarrow 4x = 240$$

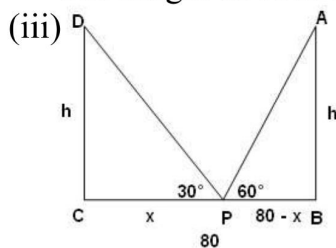
$$\Rightarrow x = 60$$

Thus, the position of the point P is 60 m from C.



$$\text{Height of the tower, } h = \frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

The height of each tower is  $20\sqrt{3}$  m.



The distance between Neeta and top of tower AB.

In  $\triangle ABP$

$$\sin 60^\circ = \frac{AB}{AP}$$

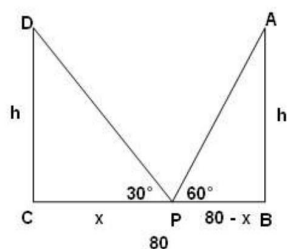
$$\Rightarrow AP = \frac{AB}{\sin 60^\circ}$$



$$\Rightarrow AP = \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow AP = 40 \text{ m}$$

OR



The distance between Neeta and top of tower CD.

In  $\triangle CDP$

$$\sin 30^\circ = \frac{CD}{PD}$$

$$\Rightarrow PD = \frac{CD}{\sin 30^\circ}$$

$$\Rightarrow PD = \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3}$$

$$\Rightarrow PD = 40\sqrt{3}$$